DETERMINATION OF THE FORCE ACTING ON A SPHERICAL OBSTACLE IN AN UNDEREXPANDED JET

G. A. Akimov

UDC 532.525:621.43.011

A method is proposed for determining the force acting on a spherical obstacle in an underexpanded jet. The method is based on the application of the momentum law.

If the pressure distribution over the surface of a body is known, then the total force acting on it is determined from the equation

$$\overline{N} = -\int_S p\bar{n}dS. \tag{1}$$

The distribution of the parameters of a gas over the surface of a body is determined with a high accuracy through the solution of the problem of the interaction of a supersonic jet with the body. Numerical solutions of several variants of this problem have now been obtained. They are all rather laborious and presume the use of a computer. A simple means of calculating the pressure of a jet on an obstacle is Newton's method. In flows containing shock waves, however, the possibilities of its application are limited.

Since one is not always able to obtain the pressure distribution at the surface of the body, and hence to make use of the dependences (1), one often uses the method of determining the total force based on the use of the momentum law. When using this law one often knows the parameters of the gas at the boundaries of a centrol region assigned in the stream. This fact complicates the obtainment of a calculating dependence for the force acting on an obstacle of finite size, since one must make certain assumptions about the character of the gas motion at those boundaries of the region where the flow parameters are unknown.

We will consider a scheme of flow when a jet interacts with a spherical obstacle whose transverse size is comparable with the diameter of the jet (Fig. 1). In this case a compression shock 5, which interacts with the "hanging" shock 3 of the free jet, forms ahead of the obstacle. A reflected shock 4 and a contact surface 6 are formed as a result. The gas passing through the "hanging" and reflected compression shocks remains supersonic, as a rule, while the gas passing through the central compression shock becomes subsonic. The reflected shock interacts with the jet boundary at the point F, producing a bend in it and spreading of the stream.

We will apply the momentum law to the region bounded by the nozzle cut AA, the free boundary AF of the jet, the annular surface FE, and the surface EO of the obstacle:

$$\int_S [\rho\bar{v}v_n + (p - p_H)\bar{n}]\, dS = 0. \tag{2}$$

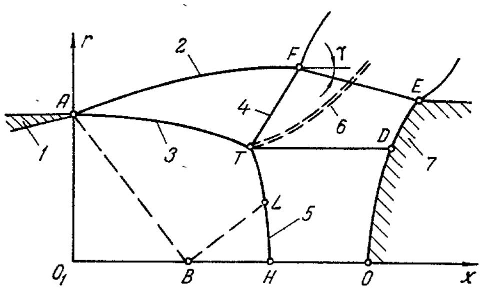
If one considers that $v_n = 0$ at the boundary of the jet and the surface of the body, and one designates the reaction of the jet on the obstacle as

$$\overline{N} = \int_{S_{EO}} (p - p_H)\, \bar{n}dS, \tag{3}$$

then one can rewrite (2) in this form:

$$\overline{N} = -\int_{S_{AA}} [\rho\bar{v}v_n + (p_a - p_H)\,\bar{n}]\, dS - \int_{S_{FE}} [\rho\bar{v}v_n + (p - p_H)\,\bar{n}]\, dS. \tag{4}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 4, pp. 668-671, October, 1978. Original article submitted December 21, 1977.



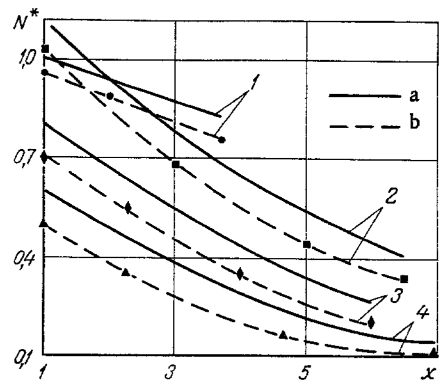


Fig. 1 Fig. 2

Fig. 1. Scheme of interaction of a jet with an obstacle: 1) nozzle; 2) jet boundary; 3) "hanging" shock; 4) reflected shock; 5) central shock; 6) contact surface; 7) obstacle.

Fig. 2. Dependence of axial force N* on distance x between nozzle and obstacle (a: calculation; b: experiment): 1) $M_a = 2.1$, $n = 2$, $\theta_a = 10°$; 2) $M_a = 2$, $n = 3.5$, $\theta_a = 10°$; 3) $M_a = 2$, $n = 4$, $\theta_a = 15°$; 4) $M_a = 2.1$, $n = 4$, $\theta_a = 15°$.

In Eq. (4) the first term equals the nozzle thrust $\overline{P}$, i.e.,

$$\overline{P} = -\int\limits_{S_{AA}} [\rho\bar{v}v_n + (p_a - p_H)\bar{n}]\, dS;$$

then we have

$$\overline{N} = \overline{P} - \int\limits_{S_{FE}} [\rho\bar{v}v_n + (p - p_H)\,\bar{n}]\, dS. \tag{5}$$

We project the latter equation onto the x axis,

$$N = P - \int\limits_{S_{FE}} [\rho v_x v_n + (p - p_H)\sin\gamma]\, dS = P - Q_a (v_x)_{av} - P_{av} S_{FE}\sin\gamma + p_H S_{FE}\sin\gamma, \tag{6}$$

where γ is the angle of inclination of the generatrix of the annular surface to the jet axis and $(v_x)_{av}$ and P_{av} are the average values of the parameters in the cross section FE, defined as the halfsum of the corresponding quantities at the points F and E. Such a definition of them is justified by the nearly linear pressure distribution in the cross section FE.

Normalizing all the quantities $Q_a v_a$ (the dynamic term of the thrust), and considering that

$$S_{FE}\sin\gamma = S_F - S_E,$$

we obtain the expression for N* in the following form:

$$N^* = 1 + \frac{1}{kM_a^2} - \frac{1}{kM_a^2 n} + \frac{\xi(M_F)\cos\theta_F + \xi(M_E)\cos\theta_E}{2\xi(M_a)} - \frac{(r_F^2 - r_E^2)(k-1)}{\varepsilon(M_a)\xi^2(M_a)4k}\left[\pi(M_F)\sigma_F - \pi(M_E)\sigma_H\right] + \frac{r_F^2 - r_E^2}{kM_a^2 n}, \tag{7}$$

where the radial coordinates are normalized to the nozzle radius at the exit.

As follows from the latter dependence, to calculate N* one must know the parameters of the jet behind the central and reflected compression shocks and the Mach number M at the point E on the surface of the sphere in addition to the flow parameters at the nozzle cut. In this connection we suggest the following calculation sequence.

1. The position of the point T on the "hanging" compression shock is assigned. (The flow in the free jet is assumed to be known.) Its vicinity is calculated.

2. The central compression shock is approximated by a quadratic parabola, the coefficients of which are determined through the parameters of the jet at the points T (the coordinates and angle of inclination of the shock) and H (the angle of inclination of the shock). The parameters at the point H are determined.

3. The reflected shock TF is approximated by a straight line.

4. The joint of intersection of the shock TF with the boundary of the jet is determined.

5. From the condition of equality of the flow rates through the central shock and the cross section TD one finds the relative position δ_{TD} of the compression shocks and the spherical obstacle,

$$\delta_{TD} = \frac{\eta q(M_a)}{r_T [q(M_T)\sigma_T \sin\theta_T + q(M_D)\sigma_H \sin\theta_D]}, \quad (8)$$

where the quantities with an index T pertain to the flow parameters on the side of the subsonic region.

The Mach number M_D is either estimated from the modified Newton's equation

$$\pi(M_D) = \sin^2\theta_D \quad (9)$$

or is found from the momentum law applied to the region bounded by the shock TH and the generatrix TD. In addition, to determine the quantity M_D one can use the condition

$$(\rho v_n)_{TD} = \text{const}.$$

All three variants of the determination of M_D give close results.

6. The Mach number M_E is determined either from Eq. (9) or with the condition that the parameters of the point E are critical. In the latter case

$$M_E = 1, \ \pi(M_E) = \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}. \quad (10)$$

7. The total force acting on the spherical obstacle is calculated from Eq. (7).

The functions $N^*(x)$ obtained by calculation and experimentally for an air jet ($k = 1.4$) flowing onto a sphere of radius $R/r_a = 2.4$ with a transverse size $r_0/r_a = 1.5$ are presented in Fig. 2. As follows from the graphs, the greatest disagreement between the experimental and calculated data does not exceed 15%.

It should be noted that the flow scheme under consideration characterizes a mode of stable interaction of the jet with the obstacle. It is known [2] that at a certain location of the obstacle in the jet the stable character of the flow is disrupted: The compression shocks oscillate ahead of the obstacle with a high frequency. The location x_* at which the unstable flow begins can be determined from the empirical dependence

$$x_* = 1.4 M_a \sqrt{kn}(1.26 - 0.17 M_a).$$

NOTATION

k	is the ratio of specific heats;
M	is the Mach number;
N	is the force of action of jet on obstacle;
n	is the degree of underexpansion;
P	is the thrust;
p	is the pressure;
Q	is the mass flow rate;
r	is the radial coordinate;
S	is the area;
v	is the velocity;
θ	is the angle of inclination of velocity vector to jet axis;
ρ	is the density;
σ	is the ratio of stagnation pressures at compression shock;
η	is the relative gas flow rate through central compression shock;
δ	is the departure of shock from obstacle;
$\varepsilon(M) = \rho/\rho_0$, $\xi(M) = v/v_{max}$, $\pi(M) = p/p_0$, $q(M) = S_*/S$	are the gasdynamic functions.

Indices

a is the nozzle cut;

0 is the stagnation parameter; other letter indices denote characteristic points of the jet.

The linear dimensions are normalized to the nozzle radius at the exit.

LITERATURE CITED

1. I. P. Ginzburg, Aerohydrodynamics [in Russian], Vysshaya Shkola, Moscow (1966).
2. B. G. Semiletenko and V. N. Uskov, "Empirical functions determining the location of shock waves in a jet flowing onto an obstacle perpendicular to its axis," Inzh.-Fiz. Zh., 23, No. 3 (1972).

WAVE STRUCTURE OF A SUPERSONIC JET DISCHARGING INTO AN OPPOSING SUPERSONIC STREAM

E. I. Sokolov and V. N. Uskov

UDC 532.525.2:533.6.011.72

A similarity parameter is suggested for the longitudinal dimensions of the wave structure of a supersonic underexpanded jet discharging into an opposing supersonic stream, and empirical equations are obtained for the calculation of these dimensions.

A number of reports devoted to the experimental investigation of the discharge of a supersonic jet into an opposing supersonic stream are presently known [1-6]. These investigations made it possible to establish the existence of two types of axisymmetric interaction of a jet with an opposing stream. If the underexpanded jet is retarded within the limits of the first barrel then an interface 1 concave to the jet (departing to infinity) and a disconnected bow shock wave 2 (type I flow) develop in the stream. Ahead of the surface, which is an impermeable barrier to the jet, a middle compression about 3 forms in the latter (Fig. 1). Near the bow surface of the body a circulation zone, closed or open depending on $P = p_{0a}/p_{T\infty}$ and $D = d_m/d_a$ develops with a pressure p_c different from p_∞ [2]. In flow of type II (penetration mode), observed with $n \sim 1$, the retardation of the jet occurs far ahead of the body, in its main section. The interaction of the jet and the stream has a nonsteady character. These types of flow are also observed in a rarefied stream [5, 6].

The wave structure formed in type I flow is analyzed below. A universal parameter of geometrical similarity of the longitudinal dimensions of the developing wave structure is suggested on the basis of the results of [1-4] and the experimental data of the authors. The presence of an infinite concave interface makes type I flow qualitatively similar to the well-studied flow when an underexpanded jet escaping into a flooded space with a pressure p_f acts on an infinite plane barrier. It is known [7] that in the case of the interaction with a barrier the introduction of the parameter $N = M_a\sqrt{kn}$ makes it possible, by using the distance h to the barrier as the characteristic dimension, to obtain an empirical dependence connecting the standoff of the middle shock formed in the jet ahead of the barrier with its location and with the discharge parameters. On the basis of the indicated qualitative analogy of the processes, we apply the complex N to the analysis of experimental data on the location of the shock waves and the interface in the discharge of a jet into an opposing stream. The range of the parameters under consideration is given in Table 1. The distance x_m to the middle shock is used as the characteristic geometrical dimension in the investigated flow. This distance is calculated from the condition of equality of the stagnation pressures on the sides of the jet and the stream at the common critical point R (Fig. 1). When the distribution of Mach numbers M along the axis of the free jet is known this condition leads to the following equations for the determination of x_m:

$$\frac{p_{0a}}{p_{s\infty}} = \left(\frac{2}{k+1}M_m^{-2} + \frac{k-1}{k+1}\right)^{\frac{k}{k-1}} \left(\frac{2k}{k+1}M_m^2 - \frac{k-1}{k+1}\right)^{\frac{1}{k-1}}, \quad M_m = M(x_m). \tag{1}$$

Leningrad Mechanics Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 4, pp. 672-676, October, 1978. Original article submitted October 19, 1977.

0022-0841/78/3504-1206$07.50